Hereditarily Optimal Realizations of Consistent Metrics

Andreas Dress¹, Katharina T. Huber², Alice Lesser³, and Vincent Moulton²

¹Max-Planck-Institut fuer Mathematik in den Naturwissenschaften Inselstrasse 22 - 26 D-04103 Leipzig, Germany
dress@mis.mpg.de

²School of Computing Sciences, University of East Anglia, Norwich, NR4 7TJ, UK
katharina.huber@cmp.uea.ac.uk, vincent.moulton@cmp.uea.ac.uk

³The Linnaeus Centre for Bioinformatics, Uppsala University, Box 598, 751 24 Uppsala, Sweden.
adice.lesser@lcb.uu.se

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Abstract. One of the main problems in phylogenetics is to find good approximations of metrics by weighted trees. As an aid to solving this problem, it could be tempting to consider optimal realizations of a metric — the guiding principle being that, the (necessarily unique) optimal realization of a tree metric is the weighted tree that realizes this metric. And, although optimal realizations of arbitrary metrics are, in general, not trees, but rather weighted networks, one could still hope to obtain a phylogenetically informative representation of a given metric, maybe even more informative than the best approximating tree. However, optimal realizations are not only difficult to compute, they may also be non-unique. Here we focus on one possible way out of this dilemma: hereditarily optimal realizations. These are essentially unique, and can be described in a rather explicit way. In this paper, we recall what a hereditarily optimal realization of a metric is and how it is related to the 1-skeleton of the tight span of that metric, and we investigate under what conditions it coincides with this 1-skeleton. As a consequence, we will show that hereditarily optimal realizations for consistent metrics, a large class of phylogenetically relevant metrics, can be computed in a straightforward fashion.

Keywords: tight span, finite metric space, optimal realization, weakly compatible

References