Fault Tolerance of Cayley Graphs

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Abstract. It is a difficult problem in general to decide whether a Cayley graph $\text{Cay}(G; S)$ is connected where G is an arbitrary finite group and S a subset of G . For example, testing primitivity of an element in a finite field is a special case of this problem but notoriously hard. In this paper, it is shown that if a Cayley graph $\text{Cay}(G; S)$ is *known* to be connected then its fault tolerance can be determined in polynomial time in $|S|\log(|G|)$. This is accomplished by establishing a new structural result for Cayley graphs. This result also yields a simple proof of optimal fault tolerance for an infinite class of Cayley graphs, namely exchange graphs. We also use the proof technique for our structural result to give a new proof of a known result on quasiminimal graphs.

Keywords: Cayley graph, fault tolerance, connectivity

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