Enumeration of Decomposable Combinatorial Structures with Restricted Patterns

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Abstract. Decomposable combinatorial structures are studied with restricted patterns. We focus on the decomposable structures in the exp-log class. Using the method of analysis of singularities introduced by Flajolet and Odlyzko [5], we provide an estimate for the probability that a decomposable structure of size n has a given restricted pattern. We exemplify with several decomposable structures like permutations and polynomials over finite fields.

Keywords: decomposable structures, restricted pattern, labeled and unlabeled structures, generating functions, exp-log class

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