

An Improvement of the Godsil Bound

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Received August 20, 1999

AMS Subject Classification: 05Exx

Abstract. In 1988, Godsil [12] showed that for a distance-regular graph with valency k and an eigenvalue with multiplicity $m \geq 3$ the diameter d is bounded by $3m - 4$. In this note, we show that $d < m + 2 + \log_5(m)$. Furthermore, we show that if the numerical girth is at least 6, then $d \leq m + 6$. Finally, we show that if the numerical girth is at least 12 then $kd < 48m$.

Keywords: distance-regular graph, diameter, multiplicity

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