Annals of Combinatorics 6 (2002) 45-56
0218-0006/02/010045-12$1.50+0.20/0

Annals of Combinatorics

Polynomial Sequences of Binomial Type and Path Integrals

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Received August 9, 1998

AMS Subject Classification: 05A40, 05A15, 58D30, 81Q30, 81R30, 81S40

Abstract. Polynomial sequences $p_n(x)$ of binomial type are a principal tool in the umbral calculus of enumerative combinatorics. We express $p_n(x)$ as a *path integral* in the "phase space" $\mathbb{N} \times [-\pi, \pi]$. The Hamiltonian is $h(\phi) = \sum_{n=0}^{\infty} p_n'(0)/n!\, e^{in\phi}$ and it produces a Schrödinger type equation for $p_n(x)$. This establishes a bridge between enumerative combinatorics and quantum field theory. It also provides an algorithm for parallel quantum computation.

Keywords: Feynman path integral, umbral calculus, polynomial sequence of binomial type, token, Schrödinger equation, propagator, wave function, cumulants, quantum computation

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