

Restricted 1-3-2 Permutations and Generalized Patterns

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Received October 19, 2001

AMS Subject Classification: 05A05, 05A15, 42C05

Abstract. Recently, Babson and Steingrímsson (see [2]) introduced generalized permutations patterns that allow the requirement that two adjacent letters in a pattern must be adjacent in the permutation. We study generating functions for the number of permutations on n letters avoiding 1-3-2 (or containing 1-3-2 exactly once) and an arbitrary generalized pattern τ on k letters, or containing τ exactly once. In several cases, the generating function depends only on k and can be expressed via Chebyshev polynomials of the second kind, and the generating function of Motzkin numbers.

Keywords: restricted permutations, generalized patterns, Chebyshev polynomials

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