

The Mix of a Regular Polytope with a Face

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Abstract. Mixing is an operation which yields subgroups generated by involutions of a larger group of the same kind. When it is applied to the product of the automorphism groups of two regular polytopes, one talks about the mix of the polytopes. This paper is concerned with conditions under which the mix of two regular polytopes is again a regular polytope. In the important special case of a mix of a polytope with one of its faces, fairly general results about the polytopality of the mix are obtained. In particular, the case when the mix is isomorphic to the original polytope is characterized.

Keywords: polyhedra and polytopes, regular figures, division of space

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