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Some Basic Extensions of Gustafson-Rakha's Multivariate Basic Hypergeometric Series

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Abstract. In this paper we extend some special cases of the multivariate basic hypergeometric series associated to the roots system of type A_m that has been established and proved in [8]. For both types of the series, we will prove that when $m = 2n$, $n = 1$ one of the series is equivalent to Jackson's ${}_8\Psi_7$ sum, while the other series is equivalent to the basic Gauss' sum.

Keywords: q-series, q-beta integrals, integral transformations, hypergeometric series very well poised on Lie algebras

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