

Second Neighborhood via First Neighborhood in Digraphs

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Abstract. Let D be a simple digraph without loops or digons. For any $v \in V(D)$, the first out-neighborhood $N^+(v)$ is the set of all vertices with out-distance 1 from v and the second neighborhood $N^{++}(v)$ of v is the set of all vertices with out-distance 2 from v . We show that every simple digraph without loops or digons contains a vertex v such that $|N^{++}(v)| \geq \gamma|N^+(v)|$, where $\gamma = 0.657298\dots$ is the unique real root of the equation $2x^3 + x^2 - 1 = 0$.

Keywords: digraph, cycle, in-degree, out-degree, neighborhood

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