

Eigenvalues of Random Power law Graphs

Fan Chung*, Linyuan Lu, and Van Vu†

Department of Mathematics, University of California, San Diego, La Jolla, CA 92093, USA
{fan, vanvu}@ucsd.edu, llu@math.ucsd.edu

Received February 28, 2003

AMS Subject Classification: 05C80

Abstract. Many graphs arising in various information networks exhibit the “power law” behavior — the number of vertices of degree k is proportional to $k^{-\beta}$ for some positive β . We show that if $\beta > 2.5$, the largest eigenvalue of a random power law graph is almost surely $(1 + o(1))\sqrt{m}$ where m is the maximum degree. Moreover, the k largest eigenvalues of a random power law graph with exponent β have power law distribution with exponent $2\beta - 1$ if the maximum degree is sufficiently large, where k is a function depending on β , m and d , the average degree. When $2 < \beta < 2.5$, the largest eigenvalue is heavily concentrated at $cm^{3-\beta}$ for some constant c depending on β and the average degree. This result follows from a more general theorem which shows that the largest eigenvalue of a random graph with a given expected degree sequence is determined by m , the maximum degree, and \tilde{d} , the weighted average of the squares of the expected degrees. We show that the k -th largest eigenvalue is almost surely $(1 + o(1))\sqrt{m_k}$ where m_k is the k -th largest expected degree provided m_k is large enough. These results have implications on the usage of spectral techniques in many areas related to pattern detection and information retrieval.

Keywords: random graphs, power law, eigenvalues

References

1. W. Aiello, F. Chung, and L. Lu, A random graph model for massive graphs, STOC 2001, pp. 171–180, Experiment. Math. **10** (2001) 53–66.
2. W. Aiello, F. Chung, and L. Lu, Random evolution in massive graphs, In: Handbook of Massive Data Sets, Vol. 2, J. Abello et al. Eds., Kluwer Academic Publishers, 2002, pp. 97–122. Extended abstract appeared in FOCS 2001, pp. 510–519.
3. R. Albert, H. Jeong, and A. Barabási, Diameter of the world wide web, Nature **401** (1999) 130–131.
4. A. Barabási and R. Albert, Emergence of scaling in random networks, Science **286** (1999) 509–512.
5. F. Chung and L. Lu, Connected components in random graphs with given expected degree sequences, Ann. Combin. **6** (2002) 125–145.

* Research supported in part by NSF Grants DMS 0100472 and ITR 0205061.

† Research supported in part by NSF grant DMS-0200357 and by an A. Sloan fellowship.

6. P. Erdős and T. Gallai, Gráfok előírt fokú pontokkal (Graphs with points of prescribed degrees, in Hungarian), *Mat. Lapok* **11** (1961) 264–274.
7. P. Erdős and A. Rényi, On random graphs I, *Publ. Math. Debrecen* **6** (1959) 290–297.
8. M. Faloutsos, P. Faloutsos, and C. Faloutsos, On power-law relationships of the internet topology, *ACM SIG-COMM '99, Comput. Commun. Rev.* **29** (1999) 251–263.
9. I.J. Farkas, I. Derényi, A.-L. Barabási, and T. Vicsek, Spectra of “Real-World” graphs: beyond the semi-circle law, *cond-mat/0102335*.
10. C. Godsil and G. Royle, *Algebraic Graph Theory*, Springer-Verlag, New York, 2001.
11. K.-I. Goh, B. Kahng, and D. Kim, Spectra and eigenvectors of scale-free networks, *cond-mat/0103337*.
12. M. Habib, C. McDiarmid, J. Ramirez-Alfonsin, and B. Reed, Eds., *Probabilistic Methods for Algorithmic Discrete Mathematics, Algorithms and Combinatorics*, **16**, Springer-Verlag, Berlin, 1998, pp. 195–248.
13. H. Jeong, B. Tomber, R. Albert, Z. Oltvai, and A. L. Barabási, The large-scale organization of metabolic networks, *Nature* **407** (2000) 378–382.
14. J. Kleinberg, S. R. Kumar, P. Raghavan, S. Rajagopalan, and A. Tomkins, The web as a graph: measurements, models and methods, *Proceedings of the International Conference on Combinatorics and Computing*, 1999.
15. L. Lu, The diameter of random massive graphs, In: *Proceedings of the Twelfth ACM-SIAM Symposium on Discrete Algorithms*, 2001, pp. 912–921.
16. M. Mihail and C. Papadimitriou, On the eigenvalue power law, preprint.
17. O. Perron, *Theorie der algebraischen Gleichungen*, II (zweite Auflage), de Gruyter, Berlin, 1933.