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Branching Processes with Negative Offspring Distributions

Ioana Dumitriu[1], Joel Spencer[2], and Catherine Yan[3*]

[1]Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139 USA
dumitriu@math.mit.edu

[2]Courant Institute of Mathematical Sciences, New York University, New York, NY 10012, USA
spencer@cims.nyu.edu

[3]Department of Mathematics, Texas A&M University, College Station, TX 77843, USA
cyan@math.tamu.edu

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Abstract. A branching process is a mathematical description of the growth of a population for which the individual produces offsprings according to stochastic laws. Branching processes can be modeled by one-dimensional random walks with non-negative integral step-sizes. In this paper we consider the random walk with a similar algebraic setting but the step-size distribution is allowed to take negative values. We gave the exact formula for the limit probability under which the random walk continues forever. Asymptotic results are also presented.

Keywords: branching processes, negative offsprings

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