

Priority Queue Sorting and Labeled Trees

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Abstract. We construct a bijection between labeled trees and allowable pairs of permutations sorted by a priority queue. These are also the pairs of permutations that avoid the pattern pairs $(12, 21)$ and $(321, 132)$. Our bijection has the additional property that it maps leaves in the tree to descents in the output permutation in the allowable pair and, more generally, preserves the edge-deletion distribution of the tree.

Keywords: priority queue, allowable pairs, labeled trees

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