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Posets of Matrices and Permutations with Forbidden Subsequences

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Abstract. The enumeration of permutations with specific forbidden subsequences has applications in areas ranging from algebraic geometry to the study of sorting algorithms. We consider a ranked poset of permutation matrices whose global structure incorporates the solution to the equivalent problem of enumerating permutations which contain a *required* subsequence. We describe this structure completely for saturated chains of lengths one and two, so settling several new and general instances of the original problem, and conclude with a superficial asymptotic investigation of arbitrary chains whose length is small by comparison with the rank of its constituent permutations. The value of this approach is reflected in the appearance of closed polynomial formulae (related to the Robinson-Schensted correspondence) and of a framework for the systematic analysis of associated combinatorial questions; indeed, we begin by studying a simpler poset of 0-1 sequences as the natural environment in which to introduce our *insertion* and *deletion* operators.

Keywords: permutations, forbidden subsequences, permutation matrices, posets, enumeration

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