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Generalized de Bruijn Cycles

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Abstract. For a set of integers $\mathcal{I}$, we define a q-ary $\mathcal{I}$-cycle to be an assignment of the symbols 1 through q to the integers modulo q^n so that every word appears on some translate of $\mathcal{I}$. This definition generalizes that of de Bruijn cycles, and opens up a multitude of questions. We address the existence of such cycles, discuss "reduced" cycles (ones in which the all-zeroes string need not appear), and provide general bounds on the shortest sequence which contains all words on some translate of $\mathcal{I}$. We also prove a variant on recent results concerning decompositions of complete graphs into cycles and employ it to resolve the case of $|\mathcal{I}| = 2$ completely.

Keywords: de Bruijn cycle, graph decomposition, probabilistic method

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