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The Problem of the Pawns

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Abstract. In this paper we study the number $M_{m,n}$ of ways to place nonattacking pawns on an $m \times n$ chessboard. We find an upper bound for $M_{m,n}$ and consider a lower bound for $M_{m,n}$ by reducing this problem to that of tiling an $(m+1) \times (n+1)$ board with square tiles of size 1×1 and 2×2. Also, we use the transfer-matrix method to implement an algorithm that allows us to get an explicit formula for $M_{m,n}$ for given m. Moreover, we show that the double limit $\nu := \lim_{m,n \to \infty} (M_{m,n})^{1/mn}$ exists and $2.25915263 \leq \nu \leq 2.26055675$. Also, the true value of ν is around $2.2591535382327 \cdots$.

Keywords: pawns, nonattacking placements, tilings, transfer matrices, asymptotic value

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