

Permutation Sign under the Robinson-Schensted Correspondence

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Abstract. We show how the sign of a permutation can be deduced from the tableaux induced by the permutation under the Robinson-Schensted correspondence. The result yields a simple proof of a conjecture on the squares of imbalances raised recently by Stanley.

Keywords: sign, permutations, tableaux, Robinson-Schensted correspondence, Knuth equivalence, Beissinger algorithm, sign-imbalance

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