A Note on a Theorem of Horst Sachs*

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Abstract. In this note, we present an apparently new and rather short proof of a celebrated theorem of Horst Sachs characterizing bipartite finite graphs in term of their eigenvalue spectrum. Moreover, the simplicity of the proof allows us to establish this theorem and related results as a special instance of much more general assertions regarding the spectral theory of “compact graphs”. Finally, some intriguing possible generalizations to locally finite, yet not “compact” graphs suggested by Horst Sachs are discussed in the last section.

Keywords: bipartite graphs, eigenvalues of graphs, compact graphs, locally finite graphs, harmonic graphs, semiharmonic graphs

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