Symmetrization of Closure Operators and Visibility

Horst Martini and Walter Wenzel

Fakultät für Mathematik, Technische Universität Chemnitz, D-09107 Chemnitz, Germany
{martini, walter}@mathematik.tu-chemnitz.de

Received March 9, 2005

AMS Subject Classification: 06A15, 05B35, 06B05, 51D20, 52A05

Dedicated to the Memory of Gian-Carlo Rota

Abstract. For an arbitrary set E and a given closure operator $\sigma: \mathcal{P}(E) \rightarrow \mathcal{P}(E)$, we want to construct a symmetric closure operator $\hat{\sigma}: \mathcal{P}(E) \rightarrow \mathcal{P}(E)$ via some – possibly infinite – iteration process. If E is finite, the corresponding symmetric closure operator $\hat{\sigma}$ defines a matroid. If $E = \mathbb{R}^n$ and σ is the convex closure operator, $\hat{\sigma}$ turns out to be the affine closure operator. Moreover, we apply the symmetrization process to closure operators induced by visibility.

Keywords: closure operator, symmetric and finitary closure operator, lattices, matroid, visibility problem, illumination problem, convexity

References

1. V. Boltyanski, H. Martini, and P. Soltan, *Excursions into Combinatorial Geometry*, Springer-Verlag, New York, 1997.
2. B.A. Davey and H.A. Priestley, *Introduction to Lattices and Order*, Cambridge University Press, Cambridge, 2002.
3. A.W.M. Dress and W. Wenzel, Matroidizing set systems — a new approach to matroid theory, *Appl. Math. Lett.* **3** (2) (1990) 29–32.
4. B. Ganter and R. Wille, *Formale Begriffsanalyse*, Springer-Verlag, Heidelberg, 1996.
5. H. Hadwiger, Ungelöste Probleme, Nr. 38, *Elem. Math.* **15** (1960) 130–131.
6. A. Higuchi, Lattices of closure operators, *Discrete Math.* **179** (1-3) (1998) 267–272.
7. R.E. Jamison, A development of axiomatic convexity, In: Technical Report no. 48, Clemson University, Math., (1970) pp. 15–20.
8. R.E. Jamison, A perspective on abstract convexity: classifying alignments by varieties, In: *Convexity and Related Combinatorial Geometry*, D.C. Kay and M. Breen, Eds., Marcel Dekker, (1982) pp. 113–150.
9. B. Korte, L. Lovász, and R. Schrader, *Greedoids*, Springer-Verlag, Berlin, 1991.
10. H. Martini and V. Soltan, Combinatorial problems on the illumination of convex bodies, *Aequationes Math.* **57** (1999) 121–152.

11. H. Martini and W. Wenzel, A characterization of convex sets via visibility, *Aequationes Math.* **64** (2002) 128–135.
12. H. Martini and W. Wenzel, An analogue of the Krein-Milman theorem for star-shaped sets, *Beiträge Algebra Geom.* **44** (2) (2003) 441–449.
13. H. Martini and W. Wenzel, Illumination and visibility problems in terms of closure operators, *Beiträge Algebra Geom.* **45** (2) (2004) 607–614.
14. K. Meyberg, *Algebra, Teil 2*, Carl Hanser-Verlag, München–Wien, 1976.
15. F.A. Valentine, Visible shorelines, *Amer. Math. Monthly* **77** (1970) 146–152.
16. R. Webster, *Convexity*, Oxford University Press, Oxford, 1994.
17. N. White, *Theory of Matroids*, Cambridge University Press, Cambridge, 1986.